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PENETRABLE WEDGE ANALYSIS
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I. PHYSICAL IMPEDANCE BOUNDARY CONDITION

We can now confidently state that the pair of coupled difference equations, that arose [61] in the Kontorovich-Lebedev analysis of the actual Leontovich-boundary-condition wedge, are a more complicated and therefore inferior version of William's [7] formulae. Although the literature has concentrated on the formulation of Maliuzhinets [6], an asymptotic evaluation of the Williams result to extract high frequency diffraction mechanisms would ultimately duplicate the work of Tiberio, et al [10], since the Williams and Maliuzhinets results are necessarily equivalent. Therefore, we will not be pursuing the physical impedance boundary wedge anymore during the remainder of this grant.

II. INHOMOGENEOUS OR LINEARLY-VARYING IMPEDANCE BOUNDARY

Felsen's 1959 (!) paper [66] presents a rather complete analysis and discussion of this curious problem that I re-discovered in 1992 [61]. A thorough dissection and understanding of Felsen's work is required before any meaningful extensions or extrapolations can be made. This has not been done to date, owing to our attention to the:

III. GENERAL PENETRABLE WEDGE

Since our previous quarterly report of 3 August 1993, and motivated partly by the findings above for both cases of impedance boundaries, we have returned (again!) to the truly penetrable wedge scatterer. A fresh, and in retrospect, logical approach starting with a fundamental application of Green's theorem, has yielded a surface integral equation for a single unknown surface distribution. The kernel is specific to the wedge geometry; the free-space Green's function is not used. The radiation condition is invoked in *both* regions, ensuring uniqueness of the integral equation solution. The unboundedness of the penetrable scatterer is thusly the very feature of the physical problem that permits this

formulation, while rendering integral equations developed [2-5,18-22,28,59] for finite bodies inefficient and effectively nonapplicable.

To extract the physical solution from this exciting and mathematically sound formulation of the boundary value problem, we are concentrating on a creative and physically-motivated approach to these issues:

(1) *Choice of a suitable basis in which to expand the surface distribution.* Convergence of the solution, as well as physical interpretation and utility, is greatly enhanced through an expansion that anticipates the actual physical behavior. In this problem of a semi-infinite domain, the expected far ($r \to \infty$) behavior on the wedge surface should be that of the

(2) *Line-source excitation of the Sommerfeld half-space problem.* All wave mechanisms that we can asymptotically pull out of this simpler geometry and account for in the wedge problem are valuable both computationally and conceptually.

References

1. A.D. Rawlins, *Diffraction by a Dielectric Wedge*, J. Inst. Maths Applics **19** (1977), 261–279.
2. S. Berntsen, *Diffraction of an Electric Polarized Wave by a Dielectric Wedge*, SIAM J. Appl. Math. **43** (1983), 186–211.
3. V. Rokhlin, *Solution of Acoustic Scattering Problems by Means of Second Kind Integral Equations*, Wave Motion **5** (1983), 257–272.
4. R.E. Kleinman and P.A. Martin, *On Single Integral Equations for the Transmission Problem of Acoustics*, SIAM J. Appl. Math. **48** (1988), 307–325.
5. K. Davey, *An Integral Approach to Electromagnetic Scattering off a Dielectric Wedge*, Electromagnetics **7** (1987), 167–183.
6. G.D. Maliuzhinets, *Excitation, Reflection and Emission of Surface Waves from a Wedge with Given Face Impedances*, Sov. Phys. Dokl. **3** (1958), 752–755.
7. W.E. Williams, *Diffraction of an E-polarized Plane Wave by an Imperfectly Conducting Wedge*, Proc. R. Soc. London Ser. A **252** (1959), 376–393.
8. T.B.A. Senior, *Diffraction by an Imperfectly Conducting Wedge*, Commun. Pure Appl. Math. **12** (1959), 337-372.
9. A.D. Pierce and W.J. Hadden, Jr., *Plane wave diffraction by a wedge with finite impedance*, J. Acoust. Soc. Am. **63** (1978), 17–27.
10. R. Tiberio, G. Pelosi, and G. Manara, *A Uniform GTD Formulation for the Diffraction by a Wedge with Impedance Faces*, IEEE Trans. Antennas Propagat. **33** (1985), 867–873.
11. K. Hongo and E. Nakajima, *Polynomial Approximation of Maliuzhinets' Function*, IEEE Trans. Antennas Propagat. **34** (1986), 942–947.
12. M.I. Herman, J.L. Volakis, and T.B.A. Senior, *Analytic Expressions for a Function Occurring in Diffraction Theory*, IEEE Trans. Antennas Propagat. **35** (1987), 1083–1086.
13. T. Griesser and C.A. Balanis, *Reflections, Diffractions, and Surface Waves for an Interior Impedance Wedge of Arbitrary Angle*, IEEE Trans. Antennas Propagat. **37** (1989), 927–935.
14. H. Uberall, *Radar Scattering from Imperfectly Conducting or Coated Perfectly Conducting Wedges and Cones*, Acta Physica Austriaca **22** (1966), 67–93.
15. T.S. Angell and R.E. Kleinman, *Scattering of Acoustic Waves by Impedance Surfaces*, Wave Phenomena: Modern Theory and Applications (C. Rogers and T.B. Moodie, eds.), Elsevier, Amsterdam, 1984, pp. 329–336.
16. J. Bach Andersen and V.V. Solodukhov, *Field Behavior Near a Dielectric Wedge*, IEEE Trans. Antennas Propagat. **26** (1978), 598–602.
17. S.Y. Kim, J.W. Ra, and S.Y. Shin, *Edge Diffraction by Dielectric Wedge of Arbitrary Angle*, Electronics Letters **19** (1983), 851–853.

18. P.V. Tret'yakov, *Integral Solutions of the Wave Equation and the Diffraction of an Arbitrary Acoustic Wave by a Wedge*, J. Appl. Maths Mechs **55** (1991), 201–205.
19. A.W. Glisson, *An Integral Equation for Electromagnetic Scattering from Homogeneous Dielectric Bodies*, IEEE Trans. Antennas Propagat. **32** (1984), 173–175.
20. E. Marx, *Integral Equation for Scattering by a Dielectric*, IEEE Trans. Antennas Propagat. **32** (1984), 166–172.
21. E. Marx, *Computed Fields Near the Edge of a Dielectric Wedge*, IEEE Trans. Antennas Propagat. **38** (1990), 1438–1442.
22. E. Marx, *Electromagnetic Scattering from a Dielectric Wedge and the Hypersingular Integral Equation*, IEEE Trans. Antennas Propagat. (1992) (submitted).
23. S.Y. Kim, J.W. Ra, and S.Y. Shin, *Diffraction by an Arbitrary-Angled Dielectric Wedge: Part I-Physical Optics Approximation*, IEEE Trans. Antennas Propagat. **39** (1991), 1272–1281.
24. ______, *Part II-Correction to Physical Optics Solution*, IEEE Trans. Antennas Propagat. **39** (1991), 1282–1292.
25. H.H. Syed and J.L. Volakis, *Equivalent Current Formulation for an Impedance Wedge of Arbitrary Included Angle*, IEEE-APS Symposium Digest (Chicago) (1992), 1857–1860.
26. H.H. Syed and J.L. Volakis, *An Approximate Skew Incidence Diffraction Coefficient for an Impedance Wedge*, Electromagnetics **12** (1992), 33–55.
27. C. Bergljung and L.G. Olsson, *A Comparison of Solutions to the Problem of Diffraction of a Plane Wave by a Dielectric Wedge*, IEEE-APS Symposium Digest (Chicago) (1992), 1861–1864.
28. E. Marx, *The Hypersingular Integral Equation and the Dielectric Wedge*, IEEE-APS Symposium Digest (Chicago) (1992), 1865–1868.
29. L.B. Felsen, *Diffraction of the Pulsed Field from an Arbitrarily Oriented Electric or Magnetic Dipole by a Perfectly Conducting Wedge*, SIAM J. Appl. Math. **26** (1974), 306–312.
30. J.B. Keller and A. Blank, *Diffraction and Reflection of Pulses by Wedges and Corners*, Commun. Pure Appl. Math. **IV** (1951), 75–94.
31. T.W. Veruttipong, *Time Domain Version of the Uniform GTD*, IEEE Trans. Antennas Propagat. **38** (1990), 1757–1764.
32. J.M. Arnold and L.B. Felsen, *Rays and local modes in a wedge-shaped ocean*, J. Acoust. Soc. Am. **73** (1983), 1105–1119.
33. A. Kamel and L.B. Felsen, *Spectral theory of sound propagation in an ocean channel with weakly sloping bottom*, J. Acoust. Soc. Am. **73** (1983), 1120-1130.
34. J.M. Arnold and L.B. Felsen, *Intrinsic modes in a nonseparable ocean waveguide*, J. Acoust. Soc. Am. **76** (1984), 850–860.
35. J.M. Arnold and L.B. Felsen, *Coupled mode theory of intrinsic modes in a wedge*, J. Acoust. Soc. Am. **79** (1986), 31–40.
36. A.P. Ansbro and J.M. Arnold, *Calculation of the Green's function for the wedge-shaped layer*, J. Acoust. Soc. Am. **90** (1991), 1539–1546.
37. J.M. Dunn, *Lateral Wave Propagation in a Wedge Shaped Region*, IEEE Trans. Antennas Propagat. **35** (1987), 947–955.
38. M.A. Biot and I. Tolstoy, *Formulation of wave propagation in infinite media by normal coordinates with an application to diffraction*, J. Acoust. Soc. Am. **29** (1957), 381–391.
39. I. Tolstoy, *Diffraction by a Hard Truncated Wedge and a Strip*, IEEE Trans. Oceanic Eng. **14** (1989), 4–16.
40. I. Tolstoy, *Exact, explicit solutions for diffraction by hard sound barriers and seamounts*, J. Acoust. Soc. Am. **85** (1989), 661-669.
41. R.S. Keiffer and J.C. Novarini, *A Wedge Assemblage Method for 3-D Acoustic Scattering from Sea Surfaces: Comparison with a Helmholtz-Kirchhoff Method*, Computational Acoustics - Volume 1 (D. Lee, A Cakmak, and R. Vichnevetsky, eds.), Elsevier, Amsterdam, 1990, pp. 67–81.
42. H. Medwin, *Shadowing by finite noise barriers*, J. Acoust. Soc. Am. **69** (1981), 1060–1064.
43. A.I. Papadopoulos and C.G. Don, *A study of barrier attenuation by using acoustic pulses*, J. Acoust. Soc. Am. **90** (1991), 1011–1018.

44. C.G. Don, *Application of a hard truncated wedge theory of diffraction to wide barriers*, J. Acoust. Soc. Am. **90** (1991), 1005–1010.
45. D. Chu, *Impulse response of density contrast wedge using normal coordinates*, J. Acoust. Soc. Am. **86** (1989), 1883–1896.
46. G.B. Deane and C.T. Tindle, *A three-dimensional analysis of acoustic propagation in a penetrable wedge slice*, J. Acoust. Soc. Am. **92** (1992), 1583–1592.
47. E.K. Westwood, *Broadband modeling of the three-dimensional penetrable wedge*, J. Acoust. Soc. Am. **92** (1992), 2212–2222.
48. D.S. Jones, *Methods in Electromagnetic Wave Propagation*, p. 718, Clarendon Press, Oxford, 1979.
49. D.S. Jones, *Acoustic and Electromagnetic Waves*, pp. 585–590, Clarendon Press, Oxford, 1986.
50. W.R. Smythe, *Static and Dynamic Electricity*, pp. 70–72, McGraw-Hill, New York, 1968.
51. K. Atkinson, *The Numerical Solution of Laplace's Equation on a Wedge*, IMA J. Num. Anal. **4** (1984), 19–41.
52. L.C. Andrews and B.K. Shivamoggi, *Integral Transforms for Engineers and Applied Mathematicians*, pp. 245–273, Macmillan, New York, 1988.
53. N.N. Lebedev, I.P. Skalskaya, and Y.S. Uflyand, *Worked Problems in Applied Mathematics*, p. 192, Dover, New York, 1965.
54. B. Davies, *Integral Transforms and Their Applications*, pp. 195–213, Springer-Verlag, New York, 1985.
55. J. Van Bladel, *Singular Electromagnetic Fields and Sources*, pp. 149–154, Clarendon Press, Oxford, 1991.
56. User's Manual, *IMSL Math/Library: Fortran Subroutines for Mathematical Applications*, pp. 767–769, IMSL Customer Relations, Houston, TX, 1989.
57. A.M.J. Davis and R.W. Scharstein, *Electromagnetic Plane Wave Excitation of an Open-Ended, Finite-Length Conducting Cylinder*, J. Electromagn. Waves Applications **7** (1993), 301–319.
58. R.W. Scharstein, *Acoustic scattering by an open-ended hard circular tube of finite length*, J. Acoust. Soc. Am. **92** (1992), 3337–3342.
59. D. Colton and R. Kress, *Integral Equation Methods in Scattering Theory*, Wiley, New York, 1983.
60. Se-Yun Kim, *Diffraction Coefficients and Field Patterns of Obtuse Angle Dielectric Wedge Illuminated by E-Polarized Plane Wave*, IEEE Trans. Antennas Propagat. **40** (1992), 1427–1431.
61. R.W. Scharstein, *Acoustic or Electromagnetic Scattering from the Penetrable Wedge*, BER Report No. 583-162, Naval Research Laboratory - Stennis Space Center Contract No. N00014-92-C-6004, The University of Alabama, Tuscaloosa, AL, 1993.
62. R.W. Scharstein, *Mellin Transform Solution for the Static Line-Source Excitation of a Dielectric Wedge*, IEEE Trans. Antennas Propagat. (to appear).
63. M.E. Ermutlu, I.V. Lindell, and K.I. Nikoskinen, *Two-dimensional Image Theory for the Conducting Wedge*, Technical Report (1992), Helsinki University of Technology, Electromagnetics Laboratory, Espoo, Finland.
64. G.B. Deane and M.J. Buckingham, *An analysis of the three-dimensional sound field in a penetrable wedge with a stratified fluid or elastic basement*, J. Acoust. Soc. Am. **93** (1993), 1319–1328.
65. L.B. Felsen and N. Marcuvitz, *Radiation and Scattering of Waves*, pp. 674-675, Prentice-Hall, Englewood Cliffs, NJ, 1973.
66. L.B. Felsen, *Electromagnetic Properties of Wedge and Cone Surfaces with a Linearly Varying Surface Impedance*, IRE Trans. Antennas Propagat. **7** (1959), S231–S243.
67. N. Latz, *Electromagnetic Diffraction by Imperfectly Dielectric Wedges*, J. Math. Anal. Appl. **43** (1973), 373–387.
68. E.E.S. Sampaio and J.T. Fokkema, *Scattering of Monochromatic Acoustic and Electromagnetic Plane Waves by Two Quarter Spaces*, J. Geophys. Res. **97** (1992), 1953–1963.
69. V. Rokhlin, *Rapid Solution of Integral Equations of Scattering Theory in Two Dimensions*, J. Comput. Phys. **86** (1990), 414–439.
70. C. Bergljung and S. Berntsen, *Diffraction of an E-polarized Plane Wave by Two Right-angle Dielectric Wedges with Common Edge*, J. Electromagn. Waves Applic. (in review).

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The continuation of this wedge scattering research is proceeding as proposed, with several significant accomplishments and findings to date. The nature of this rigorous mathematical approach to scattering problems is such that original formulation is very important and usually very slow to evolve. For example, the previous work summarized in the report [61] had several abandoned and potentially useful starts, while the finished product is substantial.

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